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Optical Frequency Comb Generation in Monolithic Microresonators

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11.1 Introduction to Optical Frequency Combs
Optical frequency combs [1–3] provide equidistant frequency markers in the infrared, visible and extreme ultra-violet [4,5]. Early work by T.W. Hänsch recognized that the periodic pulse train of a mode-locked laser [6,7], which intrinsically provides in Fourier domain an output spectrum that constitutes an optical frequency comb, can be used to measure unknown optical frequencies. An optical frequency comb is generally characterized by its two degrees of freedom, the mode spacing $f_{\text{rep}}$, which in a pulsed laser is given by the pulse repetition rate [8], as well as the carrier envelope offset frequency $f_{\text{ceo}}$ which determines the frequency offset of the comb teeth from integer multiples of the repetition rates.
In time domain, $f_{ceo}$ describes the phase slippage of the pulse envelope with respect to the carrier. Thus, any comb line of the resulting output spectrum can be expressed by $f_m = f_{ceo} + mf_{rep}$, where $m$ is an integer. Measurement and control of $f_{rep}$ and $f_{ceo}$ allows to phase-coherently link optical frequencies across the entire spectrum spanned by the comb [8,9]. The importance of spectral broadening is not evident here, but is included later on in the abstract in the context of $2f$–$3f$ and $f$–$2f$. In this case, every comb tooth is uniquely determined by the experimentally controlled quantities $f_{ceo}$ and $f_{rep}$. Therefore, any arbitrary optical frequency within the spectrum of the comb may be synthesized, or, vice versa, any given optical frequency within this spectrum may be phase-coherently compared to the frequencies $f_{ceo}$ and $f_{rep}$ in the RF domain, which can be referenced to microwave time standard [10,11]. This unique ability has made the frequency comb a revolutionary tool in metrology and laser spectroscopy, and, serving as a clockwork mechanism, has enabled the development and practical use of time standards in the optical domain [2,3,12,13]. Beyond these advances in metrology and precision measurements, frequency combs have given new impetus to applications such as broadband laser-based gas sensing [14–16] or cavity ring down spectroscopy [17].

In an earlier work [18] Kourogi generated optical frequency combs by inserting a phase modulator inside a cavity. Injection of a continuous wave laser resonant with one of the cavity mode lead to the generation of multiple sidebands when the modulator driving frequency is matched to a multiple of the cavity’s modespacing, in this case, the bandwidth of the comb was limited by the dispersion of the cavity and phase modulator. Combs generated in this way could span several terahertz. This work was refined in later research at the Joint Institute for Laboratory Astrophysics (JILA) [19,20].

Prior to this work, the ability to link an optical frequency down to the RF domain in a phase-coherent way was pursued in experiments at the Physikalisch-Technische Bundesanstalt (PTB) in Braunschweig, Germany, following an approach taken at the National Bureau of Standards (NBS, now National Institute of Standard and Technology, NIST) in Boulder, CO. Nonlinear optical processes, in conjunction with a plethora of auxiliary oscillators, were used to phase-coherently link a Cs atomic clock reference to the optical domain [21]. For the task of bridging a large frequency interval, researchers at the Max Planck Institute of Quantum Optics (MPQ) in Garching, Germany, had used another approach: in their so-called optical interval divider chain, the interval to be measured was successively divided using several identical compact semiconductor lasers, until a direct microwave signal could be measured [22].

A revolutionary and extremely powerful tool—that made frequency chains obsolete—was realized with the demonstration that the optical frequency combs from a femtosecond laser can be fully stabilized and phase-coherently compared to the RF standards. While mode-locked lasers naturally generate an optical frequency comb owing to the periodic nature of the optical emission, a key ability of a frequency comb is to stabilize the optical spectrum in terms of the absolute frequency of each comb tooth. While locking of the mode spacing of a mode-locked laser can be readily implemented, full stabilization requires measuring and locking the carrier envelope frequency ($f_{ceo}$). A powerful technique to measure the carrier envelope frequency and thereby creating an optical frequency synthesizer is the use of an $f$–$2f$ (or $2f$–$3f$) interferometer, which requires in the former case however an optical spectrum that spans a full octave. With the advent of photonic crystal fibers, that exhibited a zero dispersion wavelength in the 800 nm range, it became possible to broaden the output spectrum of a Ti:Sapphire laser system to a full octave and thereby measure (and stabilize) for the first time the carrier envelope offset frequency [10,11,13]. This marked the beginning of a new era of frequency metrology, and led to new advances in diverse fields.
such as optical frequency metrology and precision measurements and in applications such as broadband laser-based gas sensing [17] and molecular fingerprinting [14–16]. Moreover frequency combs are an integral part of a new era of atomic clocks, based on optical transitions (exhibiting very narrow transition linewidth and high frequency) such as those afforded by a single trapped $^{133}$Hg$^+$ ion [23], or Sr atoms confined in an optical lattice [24].

11.2 Frequency Comb Generation From a Monolithic Silica Microresonator

11.2.1 Physics of the Comb Generation Process

In addition to the above mentioned ways of creating optical frequency combs, an entirely novel comb generator was reported at the Max Planck Institute of Quantum Optics in 2007, which allowed a dramatic reduction in size and enabled access to ultra-high repetition rates, exceeding 40 GHz. The approach is based on $\chi^3$ nonlinearity parametric frequency conversion* in ultra-high-$Q$ optical microresonators [26]. Their high optical finesse and small mode volume make them uniquely suited for optical frequency conversion based on third-order nonlinearities [27] and has led to a significant reduction in the threshold of nonlinear optical processes.

A microscope image of the silica toroidal microcavity [26] used in this work is shown in Figure 11.1. The 75-µm diameter structure supports optical whispering gallery type modes (WGM) in which the electromagnetic field is confined by continuous total internal reflection at the cavity boundary. The micro-fabrication process proceeds as described in Ref. [26], and uses a CO$_2$-laser assisted reflow, to ensure a very low surface roughness and reduced surface scattering losses. Typical resonance quality ($Q$) factors exceed $10^8$, mainly limited by water absorption. Due to the total internal reflection, the optical mode exhibits an evanescent tail outside the silica, which is exploited to couple light in and out of the toroid by means of an evanescent coupling to a tapered optical fiber [27,28] as shown in Figure 11.1. If the position, parallelism and diameter (of the order of the light wavelength) of the fiber are carefully adjusted, one can couple light very efficiently to the optical microresonator, and in particular one can tune the coupling strength by varying the distance between the fiber taper and the rim of the toroid.

When pumping an ultra high-$Q$ ($>10^8$) toroid optical resonance with a continuous, intense (>10 mW) and monochromatic laser in the 1550-nm range, it is possible to observe in transmission of the fiber after the microcavity, a strongly nonmonochromatic comb-like spectrum (cf. Figure 11.1). The spectrum was measured using an optical spectrum analyzer, while pumping the toroid with 60 mW of power at 1550-nm, which corresponds to an intracavity intensity exceeding ~1 GW/cm$^2$. It consists of several bright emission lines, spaced by ca. 7 nm, corresponding to the cavity free spectral range (FSR) $\nu_{FSR} = c/(2\pi R n_{eff})$, where $R$ is the cavity radius ($R = 38$ µm in the shown case) and $n_{eff}$ the effective refractive index. The observed process is very efficient, with a typical conversion efficiency above 50% from the pump to the sidebands, which exhibit a high power per comb mode (ca. 1 mW) and can span over several hundreds of nanometers.

The mechanism responsible for this broadband comb generation is a cavity-enhanced cascaded four-wave mixing (FWM) process. The parametric frequency conversion is based

* This process has also been referred to as hyperparametric frequency conversion [25].
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on an FWM process mediated by the intensity dependent refractive index of silica (the Kerr nonlinearity, $n_2 = 2.3 \times 10^{-16} \text{ cm}^2/\text{W}$) that initiates the process and results in the creation of coherent signal and idler photons from two pump photons: $\nu_{pump} + \nu_{pump} \rightarrow \nu_I + \nu_S$ (see Figure 11.1b). The conservation of energy ensures that the generated photon pairs are symmetric in frequency with respect to the pump. This mechanism is resonantly enhanced by the cavity if idler, signal and pump frequencies all coincide with optical modes of the microresonator. Parametric oscillations from the pump into signal and idler sidebands had first been observed in optical microcavities in 2004, in both silica microtoroidal resonators [30] and crystalline CaF$_2$ resonators [31] with very low thresholds (tens of $\mu$W), as a consequence of the cavity enhancement. CaF$_2$ resonators are typically larger and consequently have a smaller free spectral range in the order of 10 GHz. The larger mode volume in this kind of resonators is compensated by a higher optical quality factors of more than $10^{10}$ [32].

It is important to note that this process can cascade via nondegenerate FWM among pump and first-order sideband, to produce higher order sidebands (for example $\nu_{pump} + \nu_I \rightarrow \nu_{II} + \nu_S$) as can be seen in Figure 11.1. This cascaded mechanism ensures that the frequency difference of pump and first-order sidebands $|\nu_{pump} - \nu_I| = |\nu_S - \nu_{pump}|$ is exactly transferred to all higher-order inter-sideband spacings. Thus, provided that the cavity exhibits a sufficiently equidistant mode spacing, successive FWM to higher orders intrinsically leads to the generation of phase-coherent sidebands with equal spacing, that is, an optical frequency comb (termed Kerr comb in the remaining discussion). Indeed, the process

FIGURE 11.1
Optical frequency comb generation in a microresonator. Panel (a) shows the measured comb spectrum via the output of the tapered optical fiber. The individual comb modes are spaced by the free spectral range of the cavity. The change in the baseline observed close to the pump wavelength is due to the erbium doped fiber amplifier used to enhance the launched pump power. Panel (b): Physical principle of the comb generation process. Degenerate and nondegenerate four-wave mixing allow conversion of a CW laser into an optical frequency comb. (Adapted from Del Haye, P., Schliesser, A., Arcizet, O., Wilken, T., Holzwarth, R., and Kippenberg, T. J., Nature, 450, 1214, 2007.)
can cascade over a wide frequency span, determined by the condition that the frequency of higher order converted photons must be sufficiently close to a microcavity optical resonance so that the process can still be resonantly enhanced by the optical cavity [31] (cf. Figure 11.2). Therefore, the comb generation process is naturally limited by the fact that dispersion renders the cavity modes intrinsically nonequidistant.

Dispersion in microcavities arises from two contributions resulting from material and resonator waveguide dispersion. In the case of silica microcavities these two contributions have opposite signs allowing partial compensation of dispersion in the 1550-nm window. Indeed, measurements have verified that the optical modes exhibit only around 20 MHz deviation from equidistance for the cold cavity over a 100-nm wide span. This value is comparable to the frequency bandwidth of the individual optical resonances, of the order of 10 MHz for the considered sample when overcoupled [33], which explains the broadband frequency comb generation. Note that self- and cross-phase modulation mechanisms also help to compensate the residual dispersion [30].

It is important to note that the parametric process itself could in principle produce couples of signal/idler sidebands that are only pair-wise equidistant but not mutually equidistant as required for a comb. It is thus required to verify the equidistance of the spectrum. In the next section, a measurement is described which has allowed to prove the equidistance between the different lines at the mHz level, demonstrating the generation of a frequency comb from a continuously pumped monolithic high-Q microcavity [29].

Finally, in addition to energy conservation, parametric oscillations also require angular momentum conservation to be satisfied for a rotational invariant geometry. In a microcavity of the whispering gallery type, the optical eigenmodes are also eigenstates of the angular momentum and have discrete propagation constants given by $\beta_m = m/R$ as a consequence of the periodic boundary condition ($e^{2\pi i m R} = 1$), where $m$ is the angular mode number. Consequently the angular phase matching condition is intrinsically satisfied when the signal and idler photons correspond to adjacent cavity modes, since in this case $2\beta_{\text{pump}} = \beta_j + \beta_S$. Analogous reasoning also applies to the case of nondegenerate FWM (e.g. $\beta_{\text{pump}} + \beta_j = \beta_{\text{id}} + \beta_{\text{om}}$). It is important to note that in contrast to the second order nonlinearity, where the phase matching condition has to be ensured for frequency differences in the optical range, the FWM only requires phase matching condition over the frequency range occupied by the optical modes involved in the parametric frequency conversion process.
11.2.2 Verification of the Comb Spectrum Equidistance

To prove that the emitted spectrum of the microcavity is indeed an optical frequency comb, it is necessary to verify that the generated modes are mutually equidistant. At present, the most accurate references for optical frequency differences are provided by optical frequency combs derived from mode-locked femtosecond lasers [2,3,34]. In the following, we describe two experiments in which the mode spacing of a microcavity frequency comb is compared against such a femtosecond laser frequency comb to verify the equidistance of the mode spacing [29].

11.2.2.1 Multiheterodyne Spectroscopy

One possible measurement approach is a multi-heterodyne scheme [15]. In such a measurement, two light beams both containing a certain number of optical fields oscillating at different frequencies (such as two frequency combs) are brought to interference on a beam splitter, giving rise to power modulation (a beat) of the output beams at the various difference frequencies of the incident modes. All beats can be simultaneously recorded with a single detector of sufficient bandwidth and individually analyzed in the Fourier domain. In particular, if both input fields are equidistant frequency combs, the frequencies of the resulting beats are also expected to be equidistant. The detected signal thus constitutes a frequency comb in the RF domain. Deviations from equidistance of either optical input are directly apparent in the RF domain. Using an erbium-fiber-based mode-locked femtosecond laser [35] (Menlo-Systems GmbH) as a reference frequency comb, for which equidistant mode spacing is established, it has been possible to verify the equidistance of the microcavity frequency comb (Figure 11.3).

The reference fiber frequency comb produces a spectrum with frequencies \( f_{\text{ceo}} + n f_r \), where \( f_r \) designates the repetition rate of \( \approx 100 \text{ MHz} \), \( f_{\text{ceo}} \) is the carrier envelope offset frequency and \( n \) an integer number of order \( 2 \times 10^6 \). The microcavity comb produces frequencies \( v_0 + m \Delta v \) (in integer). Adjusting the reference comb’s repetition rate such that a multiple of it is close to the microcavity comb mode spacing, i.e. \( m f_r = \Delta v \) (with an integer \( m_0 \)) entails that the \( N \) different microcavity comb lines will generate \( N \) different RF beat notes which will again be evenly spaced (if the Kerr comb is indeed equidistant), i.e. their frequencies will be \( f_0 + k \Delta f \) with \( \Delta f = (\Delta v \mod f_r) \) and \( k \) integer.

Figure 11.3 shows the experimental setup for such a measurement. In the experiment, a 1550-nm external cavity diode laser (ECDL) is coupled to a high-quality WGM in a silica toroidal microresonator. A cascade of sidebands is generated in the resonator through FWM, all of which can be coupled back into a standard single mode fiber using the same tapered fiber. Using fused couplers, the output is split into several branches to monitor the transmitted power, as well as the spectrum using an optical spectrum analyzer. One of the outputs is sent to a beat detection unit (BDU) for the purpose of comparison with the reference frequency comb [36]. In the BDU, the inputs from microcavity and reference combs are first converted to free-space beams using collimators. The two beams are then adjusted to orthogonal polarizations and spatially combined in a polarizing beam splitter (PBS), such that they exit the PBS at the same port. A \( \lambda/2 \)-retarder plate and a second PBS enforce interference between the two inputs with an adjustable power ratio of the input beams. A subsequent grating allows to extract spectral regions of interest, before sending the beam to a fast (\( \approx 200 \text{ MHz-bandwidth} \)) InGaAs-photoreceiver (Menlo-Systems FPD 510). The signal from this receiver is analyzed using a fast-Fourier transform (FFT) or an electronic spectrum analyzer. By adjusting the repetition rate \( f_r \) of the reference comb it is
possible to make the spacing $\Delta f$ of the resulting beats as small as ~1 MHz, so that all beat frequencies $f_0 + k\Delta f$ of interest fall in the window between DC and $f_r/2$.

The results for such a measurement on nine microcavity comb lines incident on a single BDU, covering a span of 50 nm or 6.5 THz, are shown in Figure 11.3c. No deviation from equidistance can be found at the level of 5 kHz, limited by the acquisition time. This proves the equidistance for all nine lines at a level of $10^{-9}$ (or $10^{-11}$ when referenced to the optical carrier). While higher resolutions and signal-to-noise ratios (SNR) would in principle be available by extending the measurement time, the mutual fluctuations of the two free-running comb generators however prevent such improvements in practice.

### 11.2.2.2 Proving the Equidistance of the Mode Spacing at the mHz Level

Verification of the equidistance with higher resolution necessitates reducing the mutual fluctuations of the microcavity and reference comb. This can be accomplished by locking the EDCL generating the microcavity comb to the reference comb using an offset lock.
To this end, a first BDU is used to detect the beat between the ECDL and the closest reference comb mode. The phase of the resulting RF-beat is compared to a stable RF-reference at the “offset frequency” \( f_0 \), obtained from a synthesizer referenced to the MPQ in-house hydrogen maser. By feeding back on the frequency of the pumping laser, the two RF signals are held in phase. On time scales longer than the feedback’s response time the frequency of the pump laser is thus rigidly connected to the closest reference frequency comb mode, with a frequency offset of \( f_0 \). The stability of the measurement is further improved by locking the repetition rate of the reference comb to around 100 MHz. These improvements enable measurements of much longer durations, up to several hundreds of seconds, without degradation of the beat signals due to mutual fluctuations. Measurement of radio frequencies on such a long timescale can be accomplished using radio-frequency counters. In order to achieve the SNR required for reliable counting, it is advantageous to detect each beat in a dedicated BDU. This allows an individual optimization of the polarization and power ratio of the input beams as well as a selection of narrow spectral regions reducing the amount of background light incident on the photoreceiver. The achieved SNR exceeding 30 dB in a 500-kHz bandwidth proved sufficient, in spite of still lacking about 26 dB to the shot-noise limit \([36]\) of \( \text{SNR}_{\text{shot}} \approx \eta_{\text{phot}} P_{\text{ref}}/h \Delta f \), detection efficiency \( \eta \approx 0.5 \), power of the reference comb mode beating with the microcavity Kerr comb mode \( P_{\text{rcm}} \approx 50 \text{ nW} \), optical frequency \( v \approx 200 \text{ THz} \) and detection bandwidth \( \Delta f \approx 500 \text{ kHz} \).

To verify the equidistance of the microcavity Kerr comb, at least three modes have to be measured simultaneously. One mode of the microcavity comb is the pump laser itself; its offset to the closest reference comb mode is locked to \( f_0 \). It is therefore sufficient to measure two more beats of the microcavity comb against the reference comb. Figure 11.4 shows the setup employed in the most accurate measurement of microcavity combs performed so far \([29]\). The two other counted beats \( f_1 \) and \( f_2 \) correspond to the microcavity comb lines which are located at five and seven FSRs from the pump laser. A measure for the deviation from a perfectly equidistant mode spacing is the difference of the average mode spacing between the three measured lines \( \varepsilon = (f_2 - f_0)/7 - (f_1 - f_0)/5 \). For reproducible results, all counters were referenced to the same hydrogen maser and triggered by a common trigger signal to ensure simultaneous measurements.

A slightly modified RF setup could even make one counter obsolete, by directly counting a frequency ratio instead of absolute counting of two radio frequencies. This was achieved by mixing the outputs of both BDU2 and BDU3 with the reference frequency \( f_0 \) in order to directly obtain \( f_1 - f_0 \) and \( f_2 - f_0 \) at the outputs of the mixers. Counting the ratio \( R = (f_2 - f_0)/(f_1 - f_0) \) allows direct calculation of the deviation \( \varepsilon \) according to \( \varepsilon = (R-7/5) \times (f_1 - f_0)/2 \), where \( f_1 - f_0 \) provides only a scale factor and needs not to be known very accurately. This method yielded slightly better results, which is attributed to latencies in the counters degrading temporal overlap of the counting intervals in spite of a common trigger signal.

Figure 11.5 illustrates the results and statistics of a series of measurements in the two-counter-configuration. The results obtained for \( \varepsilon \) were found to be normally distributed around a mean of \((-0.91 \pm 5.5) \text{ mHz} \) with a standard deviation of 322 mHz. Together with many more measurements in the ratio-counting configuration, accumulating to a total measurement time of several hours, the mean deviation from equidistance could be bracketed to \((-0.8 \pm 1.4) \text{ mHz} \), confirming the uniformity of the comb spacing to a level of \( 7.3 \times 10^{-18} \) relative to the optical carrier, and \( 5.2 \times 10^{-16} \) relative to the 2.1 THz span of the measured microcavity comb line \([29]\).
11.2.3 Dispersion in Toroidal Microresonators

Since the resonant enhancement of FWM requires the microcavity frequency comb modes to coincide in frequency with the high-quality WGM of the microresonator, the resonator’s dispersion is of crucial importance to the generation of combs [30,31,37]. In the following, the two contributions to resonator’s dispersion are assessed both theoretically and experimentally.

The first contribution to dispersion arises from the inherent variation of the free spectral range due to the resonator geometry. In the weak compression regime, it is possible to approximate the resonances of a toroid by those of a microsphere [38]. In this case the resonance frequency of the fundamental mode of a microsphere is approximately given by [39]:

$$v_n = \frac{c}{2\pi n R} \left( m + \frac{1}{2} + \eta_1 \left( \frac{m + 1/2}{2} \right)^{1/3} \right) + \ldots$$  \hspace{0.5cm} (11.1)
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where \( c \) is vacuum speed of light, \( n \) the refractive index, \( R \) the cavity radius and \( -\eta_1 \) the first zero of the Airy function (\( \eta_1 = 2.34 \)). As a consequence, the variation of the free spectral range \( \Delta N_{\text{FSR}} \) is given by

\[
\Delta N_{\text{FSR}} = \frac{c}{2\pi n R} \eta_1 \left( \frac{m + 1/2}{2} \right)^{-5/3} = -0.41 \frac{c}{2\pi n R} m^{-5/3} < 0
\]

(11.2)

Evidently, the free spectral range reduces with increasing frequency corresponding to a negative group velocity dispersion (GVD). Low frequency modes exhibit a shorter round trip time than high frequency modes (i.e. the dispersion is normal). This behavior can be understood by noting that higher frequency modes spatially move progressively closer to the cavity boundary, making the classical optical trajectory larger.

A second contribution is due to the material dispersion of the fused silica material constituting the resonator. Considering the fact that the refractive index \( n \) is actually a function of frequency (and therefore mode number \( m \)), \( n = n(m) \), the GVD of fused silica alone would lead to a FSR variation of

\[
\Delta N_{\text{FSR}} = \frac{\partial^2}{\partial m^2} \left( \frac{c}{2\pi n R} m \right) = \frac{c^2 \lambda^2}{4\pi^2 n^2 R^2} \cdot \text{GVD,}
\]

(11.3)

where \( \text{GVD} = - (\lambda/c)(\partial^2 n/\partial \lambda^2) \) is the group velocity dispersion of fused silica. This material parameter is well-known to change its sign in the 1300-nm wavelength region from...
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about $-100 \text{ ps/(nm.km)}$ at 800 nm (normal dispersion) to $+20 \text{ ps/(nm.km)}$ at 1550 nm (anomalous dispersion). The positive sign of the GVD cancels the geometric dispersion to some extent, rendering the FSR nearly constant over a wide frequency span. Figure 11.6 displays the FSR variation for an 80- and 160-$\mu$m diameter microsphere, considering both material and geometric dispersion. Importantly, a zero dispersion point close to the operating wavelength of 1550 nm occurs [29]. As finite element simulations performed for microtoroids [40] suggest shorter resonance wavelengths for the same mode number $m$, the zero dispersion point may therefore be expected to shift to shorter wavelengths in toroids as compared to spheres [41].

To measure the dispersion of toroidal microresonators, the fiber femtosecond laser frequency comb can again be employed as a frequency ruler, measuring the FSR as a function of wavelength. Such a measurement [29], performed on an 80-$\mu$m diameter toroidal resonator with a FSR of ca. 0.96 THz yielded a change of FSR from mode pair to mode pair of $\Delta F_{\text{FSR}} = 2.6 \text{ MHz}$ between 1577 nm and 1584 nm. In other words only a small amount of uncompensated positive dispersion was found. We anticipate that accurate, fast and reliable assessment of resonator dispersion in both theory and experiment will be a key ingredient to future research pertaining to the generation of microresonator frequency combs spanning an optical octave.

### 11.3 Stabilization of the Comb

The cascaded FWM process occurring in a toroidal silica microcavity has the potential for generating a broad and equally spaced optical frequency comb. However, modes of a microcavity are not a priori stable (cf. Figure 11.7), owing to size and refractive index fluctuations, which depend on temperature and intracavity power [42,43]. In order to allow its use in frequency metrology, an important prerequisite is that the generated...
comb spectrum can be controlled in terms of its two degrees of freedom (mode spacing and frequency offset). In the following sections it is shown that monolithic microresonator based frequency combs fulfill both of these requirements, allowing full control of the optical mode spectrum and generation of mode spacings in the microwave frequency range. We explain how to gain control on the frequency comb spectrum generated and describe the principle of the stabilization, its implementation and discuss the locking properties.

11.3.1 Principle

An optical comb spectrum can be entirely described by two parameters, namely the absolute frequency of one arbitrary mode and its mode spacing. In the case of a microcavity comb the pump frequency $\nu_{\text{pump}}$ is directly part of the generated spectrum, since the equidistance was checked when using both the pump beam or the corresponding comb line (i.e. before and after the microcavity). Therefore the generated spectrum can be described as: $\nu_m = \nu_{\text{pump}} + mf_{\text{rep}} m \in \mathbb{Z}$, consequently, full stabilization of the optical frequency comb requires both control of the pump frequency and the comb’s mode spacing. The latter can be achieved by identifying the pump power and frequency as two actuators.

In standard femtosecond comb generators, the control of the comb’s repetition rate can be accomplished by tuning the cavity length, in general via a piezoelectric element supporting one cavity mirror. This control of the cavity length can obviously not be implemented in the present case since there is no movable part in the monolithic microcavity. It is however possible to control the cavity optical path length by exploiting the temperature dependent refractive index of glass: changing the intracavity intensity modifies the cavity temperature through photon absorption, which in turn affects the refractive index. In the following section it is described how this locking technique is implemented relying on the thermorefractive effect. Note that the different additional nonlinear mechanisms that define the actual mode spacing of the Kerr comb (e.g. self/cross phase modulation [30]) also contribute to its intensity dependence.

**FIGURE 11.7**
Temporal variation of the signal and idler sidebands and their frequency difference measured on two different Beat Detection Units (5 s gate time), while the pump frequency is locked on the fiber reference comb. The two comb lines remain equidistant, but drift in time.
11.3.2 Implementation

The setup used in this stabilization experiment (cf. Figure 11.8) is similar to the one described in the previous section, except that the second beat note is now used to derive an error signal, fed back in order to control the pump intensity. The latter is achieved with a tunable gain erbium doped fiber amplifier (EDFA).

In such a configuration, the signal used to control the repetition rate is not directly proportional to the comb mode spacing since it is also affected by the pump frequency drifts. However since the latter is independently locked on an optical reference (one mode of the fully-stabilized fiber comb), one can perfectly use the second beat for testing the viability of the locking principle. In the following, it is shown that it is possible to directly measure the comb repetition rate and use it for locking.

To create an error signal suitable for locking the mode spacing, the beat generated on BDU2 (see Figure 11.8) is compared to a reference signal with a phase detector whose output serves as an error signal. It is then amplified, filtered and injected on the current input of the 980-nm diode laser pumping the EDFA, without affecting the 1550-nm pump frequency, as a direct feedback on the pump laser current would have induced. Once locked, one can tune the mode spacing by simply modifying the frequency of the reference signal.

Figure 11.8b represents the time dependence of the beat between the Kerr comb's 7th sideband with the respective reference comb line, counted with a 1-s gate time and expressed in equivalent mode spacing, by dividing the measured fluctuations by a factor of 7. During the first 65 min, the lock is disabled and one thus simply measures the free drift of the mode spacing. It exhibits fluctuations on a 5-s timescale (see inset) which is attributed to
a change in the pump intensity and coupling strength and longer time drifts due to environment temperature evolution. After 65 min, the lock is activated, and one can observe a net suppression of the mode spacing fluctuations, without affecting the pump frequency lock. This behavior can also be observed on an independent out-of-loop signal originating from the beat of the 5th sideband of the Kerr comb with the reference comb. This shows the potential of the technique described to gain control over the entire the Kerr comb. Note that during the experiments, the microcavity remains locked to the pump frequency, in particular because of the so called thermal “self-locking mechanism” [45] which will be explained later in the chapter.

### 11.3.3 Characterization of the Locking Mechanism

We characterized the stability of the comb by counting simultaneously the three beats described in Figure 11.8, namely the two in-loop signals of the pump frequency and the mode spacing locks and the out-of-loop beat 3. When recording simultaneously the beats with a 1-s gate time the two in-loop signals displayed fluctuations with a root-mean-square deviation of the order of 1 mHz, which demonstrates the efficiency of the actuators. The out-of-loop measurement of the 5th sideband presents higher noise, at the 1-Hz level, nevertheless clearly demonstrating that the two locks used in this experiment allow to gain control over the full comb spectrum. This excess of noise has different origins, one of which being only coarsely adjusted feedback parameters.

Subsequently, the tunability properties of the stabilized Kerr comb were studied. In particular the dependence of the mode spacing on the pump intensity was tested. To this end, the pump laser is brought close to one of the microcavity resonances. The high optical power used in the experiment induces significant thermal bistability. When the pump frequency approaches the microcavity resonance from the blue side, the intensity circulating inside the toroid increases and the induced heating expands the cavity effective length which makes the cavity resonance being pushed away from the laser frequency. This mechanism is responsible for a so-called “thermal self-locking” [45] of the microcavity onto the laser frequency and can be exploited for keeping the microcavity quasi resonant with the pump frequency over several tens of GHz, the effective detuning remaining of the order of the cold cavity linewidth when no other nonlinear mechanism is involved. This broad tunability of the pump frequency enables the locking of the pump laser on one of the reference comb modes (spaced by 100 MHz), without losing the microcavity optical resonance.

Once the comb is generated, the aforementioned thermal self locking also allows variation of the pump power without losing the microcavity resonance nor terminating comb generation. We subsequently measured the influence of the pump power on the comb repetition rate. The result is reproduced in Figure 11.9. It was possible to scan the repetition rate over a 5 MHz range by changing the pump power $P_{\text{pump}}$ by ca. 3 mW around a mean value of approximately 80 mW. Due to the approximately linear response, the comb mode spacing can be described by the equation: $f_{\text{rep}} = f_{\text{rep}}^0 + \gamma P_{\text{pump}}$ with an experimentally determined $\gamma = -196 \text{ kHz/mW}$, $f_{\text{rep}}^0$ being the extrapolated cold cavity repetition rate. In general, the linear parameter $\gamma$ depends on the pump frequency since the pump laser is slightly blue detuned from the cavity resonance, and the way the pump power changes affect the intracavity intensity obviously depends on the laser to cavity detuning. It would thus be more appropriate to write $\gamma(V_{\text{pump}})$. However in practice the effective detuning of the laser with respect to the cavity resonance is small, such that a change in the pump laser frequency does not affect the intracavity intensity significantly. Indeed it has been
experimentally measured that a 1 GHz change in the pump frequency linearly affects the mode spacing by only 1 MHz.

Next, the origin of the intensity dependence of the mode spacing was investigated. From the microtoroid’s high quality factors and small mode volumes, several nonlinear mechanisms can contribute to this effect by changing its geometric and refractive properties. It has been shown that the cavity static and low frequency response (below 10 kHz) to an intensity change is governed by the absorption-induced heating and concomitant refractive index change and thermal expansion, which are dominant compared to the Kerr or radiation pressure contributions [46]. The corresponding variation of the microcavity’s repetition rate due to a temperature change \( \Delta T \) can be written:

\[
\frac{\Delta f_{\text{rep}}}{f_{\text{rep}}} = (\alpha_t + \alpha_n) \Delta T.
\]

The comparison of \( \alpha_n = 1.0 \times 10^{-5} \text{K}^{-1} \), the thermorefractive coefficient of silica, to \( \alpha_t = 5.5 \times 10^{-3} \text{K}^{-1} \) its thermal expansion coefficient, allows to conclude that the main contribution arises from the temperature dependence of the refractive index. As a consequence, a 0.1 K change in the toroid temperature induces a 400 kHz change of the repetition rate.

The same nonlinear mechanism is also responsible for the optical bistability mentioned above, observed when scanning the microcavity resonance with the laser. The frequency of the bistability turning point depends on the injected power, its dependence on the pump power is represented in Figure 11.9b. It exhibits a characteristic linear behavior with a slope of \( \Gamma = -93 \text{ MHz/mW} \). This measurement was taken by scanning the laser frequency sufficiently slowly such that thermal equilibrium of the toroidal cavity was reached for each detuning value. It is possible to estimate the toroid thermalization time with the characteristic diffusion time of a thermal wave from the center of the optical mode to the toroid.
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... (approximately $\delta = 3 \mu m$ away), as given by: $\tau = \delta^2 / D = 10 \mu s$ where $D = 9.5 \times 10^{-7} m^2/s$ is the heat diffusion coefficient of silica.

The change in the toroid temperature modifies the effective refractive index $n_{eff}$ but differently affects the individual cavity resonances: $\delta n_{eff} = -m \delta n_{eff}$ for large $m$. Therefore the thermally induced nonlinearity will affect the mode spacing by a quantity $\delta v_{FSR} = \delta n_{eff}$. The bistable turning point induced by the thermal displacement $\delta n_M$ of the pumped cavity mode ($m = M$) is then expected to show a pump power dependence a factor $M$ higher than the mode spacing static dependence $\delta v_{FSR}$ if the same mechanism is responsible for the two effects. In practice we pump the $M=500^{th}$ cavity mode ($v_{pump} = 200$ THz and FSR = 400 GHz), and measured a ratio $\Gamma / \gamma = 474$. The good agreement between these numbers indicates that one can mainly explain the control gained over the comb mode spacing by the contribution of the thermal dependence of the refractive index. However several of the previously mentioned nonlinear mechanisms, such as self/cross phase modulation among the different comb lines (which only affect the oscillating modes) also contribute to the observed intensity dependence and are under current investigation. As mentioned before, the input power dependence of the mode spacing $\gamma$ also depends on the pump frequency detuning, and the previous treatment is valid only when the pump frequency is sufficiently close to the cavity resonance. As explained before, the comb mode spacing presents a strong dependence on the pump power and one can even generate with the same microcavity comb spectra with higher-order mode spacings, corresponding to multiples of the cavity FSR (cf. Figure 11.10). As a conclusion, even if the exploited mechanism still has to be completely clarified, it is in practice sufficiently nonparallel to the pump frequency actuator to allow a full stabilization of the Kerr comb.

11.3.4 Actuation Properties

An important figure of merit for a feedback loop is its actuation bandwidth. The feedback mechanism we used relies on thermal effects which are very fast, of the order of $10 \mu s$, due to the toroid micron-scale dimensions. To characterize the dynamical properties of the locking mechanism, the reference signal used to phase lock the second beat signal to, was frequency modulated (it is also possible to add an in-loop perturbation, see Figure 11.9c). When changing the reference signal frequency, the servo regulates the power sent into the microcavity to adjust the repetition rate of the Kerr comb. It was then possible to dynamically tune the repetition rate over a MHz range while remaining locked. For a higher tuning range, it is also possible to change the toroid temperature, the measured $2 GHz/K$ shift of the optical resonance at room temperature corresponds to a $-4 MHz/K$ change for the mode spacing. Such a performance is on the same order of magnitude as the actuation range of the conventional fiber frequency comb.

In order to test the actuation bandwidth, the reference signal was dithered with a signal of constant amplitude, swept in frequency. By simultaneously measuring the correction signal, the frequency range over which the implemented lock is efficient was determined. The results measured with a network analyzer are reported in Figure 11.9d. The correction signal displays a flat response up to ca. 10 kHz, implying that the lock is sufficiently robust for compensating the injected perturbation, while at higher frequencies the lock’s ability to compensate the external perturbation diminishes. The measured cutoff is of the same order of magnitude as the characteristic thermal geometric cutoff frequency given by

---

† Note however that the specific geometry of the toroid induces a more complex thermal response than this simple estimation.
Ω_c = D/Δ^2 = 2π × 16 kHz, and is also comparable to the efficiency range of the locking electronics that we used. Thus the above measurements have revealed that the locking technique has the potential to reach high frequencies of actuation (up to 10 kHz), which is on par with the actuation range of standard comb generators based on piezo-mounted mirrors.

### 11.4 Generation of a Stabilized Microwave Repetition Rate Frequency Comb

An important prerequisite for an optical frequency comb to be suitable for metrology is their ability to establish a link from the optical to the RF domain. The latter necessitates repetition rates in the microwave or RF range, amenable to direct electronic detection. In this section we present results on microwave repetition rates obtained with monolithic comb generators which employ larger, mm-scale microcavities. Note that the achieved microwave repetition rates have so far not been reached by femtosecond laser based combs [47], and can prove useful in a variety of applications, such as astronomical spectrometer calibration, and systems where the individual addressing of the comb lines plays an important role, like pulse shaping.
11.4.1 Monolithic Frequency Comb Generators with Microwave Repetition Rate

Aiming at producing microcavities having a repetition rate below 100 GHz, at frequencies which can be directly measured using fast telecom photodiodes, requires a cavity diameter of more than 660 μm. Taking into account the diameter reduction occurring during the reflow process, where the CO₂ laser evaporates part of the silica [26], the usual micro-fabrication process was followed for creating 1-mm diameter disks. The reflow procedure had to be adapted, since the laser intensity was not sufficient when trying a direct and single shot melting of the disk because of the necessary defocusing needed to cover the full disk surface with the laser. We opted for a different procedure where the CO₂ laser was strongly focused and manually scanned around the periphery of the disk. This procedure allowed fabrication of millimetric sized toroids, shown in Figure 11.11, at the expense of a stronger diameter reduction since the laser illumination was increased.

The lower part of Figure 11.11 represents the transmission of the optical fiber when the laser frequency is scanned over several free spectral ranges of the microcavity. Several classes of modes are evident, corresponding to the higher order transverse modes. The mode spacing of the 750-μm diameter cavity corresponds to 86 GHz. The Q-factor for these samples (which was not optimized yet) reached values of $2 \times 10^7$. While this value is more...
than 10 times lower than what has been observed in smaller toroidal microcavities (and 
>100 times lower than in crystalline microresonators), it was sufficient to observe paramet-
ric oscillations at a threshold value of ca. 10 mW.

Using a 750-µm diameter sample it was possible to generate a frequency comb with a
86 GHz repetition rate, by pumping the toroid at 1550 nm with a 150-mW continuous-wave
laser. The typical optical spectrum measured after the cavity is represented in Figure 11.12a.
The comb spans over 60 nm which is lower than what was demonstrated with smaller cavi-
ties [29]. This is a consequence of the increased dimensions of the toroid, which imply a
larger minimum threshold for parametric oscillations [30]:

\[ P_{\text{thres}}^{\text{min}} = \frac{Q_0^2}{K} \left(1 + K\right)^3 \pi^2 n_{\text{eff}}^2 \frac{R A_{\text{eff}}}{\lambda_0 n_2} \]

(11.4)

The expression presents a linear dependence in the effective mode volume \(2\pi R A_{\text{eff}}\)
\((K = Q_0/Q_\text{ex} \text{ is the coupling parameter, } Q_0 \text{ and } Q_\text{ex} \text{ the intrinsic and external (related to}
the coupling) optical quality factor, and } n_2 \text{ the intensity dependent refractive index).}
For the 750-µm cavities, the diameter is responsible for a ten-fold increase in thresh-
old, and the effective mode area is also expected to increase from the higher inner
toroid diameter. Those effects directly increase the threshold and similarly affect the
strength of the induced nonlinearities. Note that there is no fundamental limit which
prevents higher \(Q\) factors. Indeed, \(Q > 10^9\) have been measured in millimeter sized
microspheres [48].

11.4.2 Stabilization and Characterization of a Microwave Frequency Comb

For the 750-µm diameter samples, we verified that the comb emission allowed direct
measurement of the repetition rate. To this end, telecom detectors from u² Photonics
were employed in conjunction with a harmonic mixer fed with a ca. 14.3 GHz signal

![FIGURE 11.12](image)
(a) Typical optical spectrum measured after the millitoroid. The comb exhibits a 83 GHz comb spacing whose intensity beat can be directly measured using a fast photodiode. (b) Locked microwave beat (measured by downconverting the signal to 30 MHz using a harmonic mixer) 83 GHz for the new figure. The width of the coherent spike is determined by the resolution bandwidth of the measurement (10 Hz). (From Del’Haye, P., Arcizet, O., Schliesser, A., Holzwarth, R., and Kippenberg, T. J., Full Stabilization of a microresonator-based optical frequency comb. Phys. Rev. Lett., 101, 053903, 2008.)
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(a subharmonic of the 86 GHz signal to be measured) in order to down-convert the signal to countable frequencies, of the order of 30 MHz.

In order to use the beat signal for locking the comb, it was phase compared with a reference signal and the error signal provided was similarly used as feedback to the optical amplifier pump current. The setup is described in Figure 11.13. The technique allowed direct locking of the repetition rate of the comb without relying on the fiber reference comb. The typical locked 86 GHz signal is represented in Figure 11.12b, revealing the coherent spike and the ca. 10 kHz wide noise reduction accomplished by the feedback loop. This noise pedestal reflects the cavity optical path fluctuations above the frequency range where the lock is compensating. Many mechanisms contribute to this noise [42,49], in particular the laser’s classical frequency noise, which is not distinguishable in this measurement, the laser’s shot noise, the thermally and optically induced noise processes that affect the microcavity’s optical path length through its refractive index, expansion and higher frequency mechanical vibrations. Note that even if the opto-mechanical properties of the millitoroid have been intentionally deteriorated, by maintaining a very small undercut and leaving it in air for providing viscous damping that decreased its mechanical Q factor, the optomechanical instability [50,51] i.e. radiation pressure driven regenerative mechanical oscillations, is however very easily observed when the laser detuning is significant, and in general prevents from generating a stable optical comb (for a laser strongly blue detuned). In addition, to limit this optomechanical instability, the fiber was in general intentionally placed in contact to the microcavity for suppressing these radiation-pressure-induced mechanical oscillations.

The long term stability of the microwave beat was investigated using an RF counter, the Allan deviation measurements for the free running beat and the locked signal are shown in Figure 11.13b. The influence of the lock is observable, dramatically increasing the long term stability of the beat and yield
ing a repetition rate stability at the level of 10 mHz in 1 s, corresponding to an Allan deviation of $10^{-12}$ for the in-loop measurement. This number reflects the large stability achieved on the optical path length of the toroid. The out-of-loop measurements implemented on the first idler sideband yielded rms fluctuations $\Delta v$ below 1 Hz for a 1 hour measurement with a 1-s gate time [44]. The measured fluctuation
expressed in an equivalent fluctuation of cavity optical path length, gives a relative stability of $\Delta L_{\text{opt}}^{2} \propto P \Delta R \Delta N / N^{1.0}$ am in 1 s. The demonstrated capacity to actively control the resonator effective optical length, combined with their intrinsic stability, could allow microcavities to become a future promising tool for time transfer. [43].

11.5 Conclusion

In summary, this chapter has introduced a radically different frequency comb generator approach, unique in several ways. First, the approach offers unprecedented form factor, allowing a monolithic approach to optical frequency comb generation. Due to the high Q-factor in conjunction with tapered optical fiber coupling, the approach is moreover highly efficient, with conversion efficiencies frequently observed above 80%. Second, the approach does not rely on the use of atomic or molecular resonances, and therefore can achieve broadband gain in a frequency window only limited by material absorption or dispersion. Owing to the large transparency window of glass, the frequency window extends from the UV to 2 µm. Third, the use of monolithic microresonators allows to significantly augment the repetition rate. While the highest repetition rates of conventional femtosecond lasers so far are approaching 10 GHz, the equivalent cavity length of 3 cm is difficult to reduce further. Moreover, high repetition rate combs have so far only been achieved in Ti:Sa laser systems, whereas for 1.5 µm fiber based frequency comb repetition rates have been generally much lower. The present approach can operate at previously unattainable repetition rates exceeding 10 GHz, and with a high power per individual comb mode, which is useful in applications where the access to individual comb modes is required, such as optical waveform synthesis, high capacity telecommunications, spectrometer calibration [52] or direct comb spectroscopy. Finally, several new prospects exist that bode well for future work and for the future of this technology.

Since this work, the generation of frequency combs in this manner has also been reported in CaF$_2$ resonators [49]. This work allowed reducing the repetition rate to
10 GHz and moreover showed the promise of this technology to serve as low phase noise microwave oscillator. These developments show that microresonator-based frequency comb generation bear promise for the future, possibly providing a route to all chip scale atomic clocks.

Acknowledgments

We thank T. W. Hänsch for discussions and suggestions. T. J. K. acknowledges support via an Independent Max Planck Junior Research Group. This work was funded as part of a Marie Curie Excellence Grant (RG-UHQ) and the DFG-funded Nanosystems Initiative Munich (NIM). We thank J. Kotthaus for access to clean room facilities for sample fabrication.

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